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9. Najděte střed tětivy vyťaté přímkou p: $x + 3y - 12 = 0$ na kuželosečce

$$k:\quad 2x^2 + 4xy + 3y^2 - 3x - 3y = 0.$$

$$p\colon x = 12 - 3y$$

$p \cap k$:

$$\begin{aligned}2(12-3y)^2 + 4(12-3y)y + 3y^2 - 3(12-3y) - 3y &= 0\\ 9y^2 - 90y + 252 &= 0\\ y^2 - 10y + 28 &= 0\end{aligned}$$

$$D = 100 - 4\cdot 28 = -12$$

$$y = \frac{10 \pm \mathrm{i}2\sqrt{3}}{2} = \begin{cases} y_1 = 5 + \mathrm{i}\sqrt{3} & x_1 = -3 - 3\mathrm{i}\sqrt{3}\\ y_2 = 5 - \mathrm{i}\sqrt{3} & x_2 = -3 + 3\mathrm{i}\sqrt{3}\end{cases}$$

$$\begin{aligned}T_1 &= \left[-3 - 3\mathrm{i}\sqrt{3}, 5 + \mathrm{i}\sqrt{3}\right]\\ T_2 &= \left[-3 + 3\mathrm{i}\sqrt{3}, 5 - \mathrm{i}\sqrt{3}\right]\\ S &= \frac{T_1 + T_2}{2} = [-3, 5]\end{aligned}$$

10. Pro kuželosečku k určete normální tvar rovnic, afinní typ, normovaný polární afinní repér a transformace afinních nehomogenních souřadnic, které převádějí danou rovnici kuželosečky na normální tvar.
a)

$$k:\quad x^2 - 6xy + 9y^2 - 12x + 14y - 9 = 0$$

$$A = \begin{pmatrix} 1 & -3 & -6\\ -3 & 9 & 7\\ -6 & 7 & -9\end{pmatrix}\qquad \begin{aligned}&|A| = -81 + 126 + 126 - 324 - 49 + 81 = -121 \neq 0 \Rightarrow \text{ regulární}\\ &|\bar{A}| = 9 - 9 = 0 \Rightarrow \text{ parabolický typ}\end{aligned}$$

Afinní typ: parabola
Střed:

$$\left(\begin{array}{ccc|c} 1 & -3 & -6 & 0\\ -3 & 9 & 7 & 0\end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -3 & -6 & 0\\ 0 & 0 & -11 & 0\end{array}\right) \Rightarrow (3,1,0) \text{ je nevlastní střed } k$$

Bod na kuželosečce:

$$x^2 - (6y + 12)x + (9y^2 + 14y - 9) = 0$$
$$\text{zvolme } 6y + 12 = 0 \Rightarrow y = -2\colon\quad x^2 - 1 = 0$$

$O = [1, -2] \in k$

Tečna v O:

$$t\colon\quad \begin{pmatrix}1 & -2 & 1\end{pmatrix}\begin{pmatrix}1 & -3 & -6\\ -3 & 9 & 7\\ -6 & 7 & -9\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=0$$

$$t\colon\quad x-14y-29=0$$

$\mathbf{u}_1=(14,1)$
$\mathbf{u}_2=(3,1)$
$\langle O,\mathbf{u}_1,\mathbf{u}_2\rangle$:

$$k\colon\quad F(\mathbf{u}_1){x'}^2+2f(\mathbf{u}_2,O)y'=0$$

$$F(\mathbf{u}_1)=\begin{pmatrix}14 & 1 & 0\end{pmatrix}\begin{pmatrix}1 & -3 & -6\\ -3 & 9 & 7\\ -6 & 7 & -9\end{pmatrix}\begin{pmatrix}14\\ 1\\ 0\end{pmatrix}=\begin{pmatrix}11 & -33 & -77\end{pmatrix}\begin{pmatrix}14\\ 1\\ 0\end{pmatrix}=121$$

$$f(\mathbf{u}_2,O)=\begin{pmatrix}3 & 1 & 0\end{pmatrix}\begin{pmatrix}1 & -3 & -6\\ -3 & 9 & 7\\ -6 & 7 & -9\end{pmatrix}\begin{pmatrix}1\\ -2\\ 1\end{pmatrix}=\begin{pmatrix}0 & 0 & -11\end{pmatrix}\begin{pmatrix}1\\ -2\\ 1\end{pmatrix}=-11$$

$\langle O,\mathbf{u}_1,\mathbf{u}_2\rangle$:

$$k\colon\quad 121{x'}^2-22y'=0$$

$$\mathbf{e}_1=\frac{1}{\sqrt{121}}\mathbf{u}_1=\left(\frac{14}{11},\frac{1}{11}\right)\qquad \mathbf{e}_2=-\frac{1}{11}\mathbf{u}_2=\left(-\frac{3}{11},-\frac{1}{11}\right)$$

Normální tvar rovnice kuželosečky k v normovaném polárním afinním repéru $\langle O,\mathbf{e}_1,\mathbf{e}_2\rangle$:

$$k\colon\quad {x''}^2+2y''=0$$

Transformační rovnice:

$$\begin{aligned}x&=\frac{14}{11}x''-\frac{3}{11}y''+1\\ y&=\frac{1}{11}x''-\frac{1}{11}y''-2\end{aligned}$$

b)

$$k\colon\quad 5x^2-4xy+y^2-2x+4y-1=0$$

$$A=\begin{pmatrix}5 & -2 & -1\\ -2 & 1 & 2\\ -1 & 2 & -1\end{pmatrix}\qquad \begin{aligned}&|A|=-5+4+4-1-20+4=-14\neq 0\ \Rightarrow\ \text{regulární}\\ &|\bar{A}|=5-4=1>0\ \Rightarrow\ \text{eliptický typ}\end{aligned}$$

Střed:

$$\left(\begin{array}{ccc|c}5 & -2 & -1 & 0\\ -2 & 1 & 2 & 0\end{array}\right)\sim\left(\begin{array}{ccc|c}-2 & 1 & 2 & 0\\ 0 & 1 & 8 & 0\end{array}\right)\Rightarrow(-3,-8,1)$$

$S=[-3,-8]$
Zvolme $\mathbf{u}_1=(1,0)$
Polára $(1,0,0)$:

$$p\colon\quad \begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}5 & -2 & -1\\ -2 & 1 & 2\\ -1 & 2 & -1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=0$$

$$p\colon\quad 5x-2y-1=0$$

$\mathbf{u}_2=(2,5)$

$\langle S, \mathbf{u}_1, \mathbf{u}_2 \rangle$:

$$k\colon \quad F(\mathbf{u}_1){x'}^2 + F(\mathbf{u}_2){y'}^2 + F(S) = 0$$

$$F(\mathbf{u}_1) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 5 & -2 & -1 \\ -2 & 1 & 2 \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 & -2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 5$$

$$F(\mathbf{u}_2) = \begin{pmatrix} 2 & 5 & 0 \end{pmatrix} \begin{pmatrix} 5 & -2 & -1 \\ -2 & 1 & 2 \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 8 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} = 5$$

$$F(S) = \begin{pmatrix} -3 & -8 & 1 \end{pmatrix} \begin{pmatrix} 5 & -2 & -1 \\ -2 & 1 & 2 \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} -3 \\ -8 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -14 \end{pmatrix} \begin{pmatrix} -3 \\ -8 \\ 1 \end{pmatrix} = -14$$

$\langle S, \mathbf{u}_1, \mathbf{u}_2 \rangle$:

$$k\colon \quad 5{x'}^2 + 5{y'}^2 - 14 = 0$$

$$k\colon \quad \frac{5}{14}{x'}^2 + \frac{5}{14}{y'}^2 - 1 = 0$$

$$\mathbf{e}_1 = \sqrt{\frac{14}{5}}\mathbf{u}_1 = \left(\frac{\sqrt{70}}{5}, 0\right) \qquad \mathbf{e}_2 = \sqrt{\frac{14}{5}}\mathbf{u}_2 = \left(\frac{2\sqrt{70}}{5}, \sqrt{70}\right)$$

Normální tvar rovnice kuželosečky k v normovaném polárním afinním repéru $\langle E, \mathbf{e}_1, \mathbf{e}_2 \rangle$:

$$k\colon \quad {x''}^2 + {y''}^2 - 1 = 0$$

Afinní typ: reálná elipsa
Transformační rovnice:

$$\begin{aligned} x &= \frac{\sqrt{70}}{5}x'' - \frac{2\sqrt{70}}{5}y'' - 3 \\ y &= \sqrt{70}y'' - 8 \end{aligned}$$